Comparison of Maximum Likelihood and Time Frequency Approaches for Time Varying Delay Estimation in The Case of Electromyography Signals

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ABSTRACT

Muscle fiber conduction velocity (MFCV) is based on the time delay estimation between electromyography recording channels. In order to take into account the variability of the MFCV, we assume that the time delay between the channels varies over time. In the present paper, the Maximum - Likelihood estimation (MLE) of time varying delay for two channels of EMG signals that follow a polynomial model is derived. Monte Carlo simulations are performed at different noise levels in order to evaluate the noise impact of the estimator. The Maximum-Likelihood estimation was achieved by the Newton method. The delay with unknown model (inverse sinusoidal) was also investigated by cutting this delay into many slices. This approach gives the best results by comparison with the other ones

Keywords: conduction velocity, EMG, multichannel acquisition, fatigue, time-varying delay estimation.

1. INTRODUCTION

Muscle fiber conduction velocity (MFCV) is a relevant neuromuscular indicator of neuromuscular pathologies [8], fatigue [8], or pain [4]. MFCV can be estimated from intramuscular or surface electromyography recordings [8]. In this work, we are only interested in surface EMG signals (sEMG). The sEMG signal suffers from several limitations due to anatomical problems and changes in the action potential volume conductor that impact the conduction velocity estimation. This is particularly the case in dynamic muscle contraction conditions (the most applied daily conditions); in which both force and segment position vary. In that case, three main factors affect the sEMG signal: firstly the non-stationary property of the data; secondly the change in conductivity properties of the tissues separating electrodes and muscle fibers and finally the relative shift of the electrodes with respect to the origin of the action potential.

The first factor (non-stationary) has been investigated in [4] by considering models of time- varying delay between stationary EMG sources. This work is still limited to the two-channel case. The last two factors have been taken into account by using a multichannel device. It has been shown in [4] that using more than two surface EMG derivations along the fiber direction may significantly improve MFCV estimates in terms of estimation variance, measure sensitivity to electrode location, and repeatability.

In a recent paper [7], constant delays between two channels were investigated by the Generalized Cross-Correlation Method (GCC) [1]. This general estimation framework involving pre-filters is supposed to improve estimation results but it requires the prior knowledge of power spectra of noise and signals. In the case of real data, these power spectra have to be estimated. In present paper, the time varying delay (TVD) estimation will be investigated but still limited in the case of two channels.

The problem of TVD estimation has been previously studied by many authors [5] ,[6] ,[9]. Time-frequency/time-scale coherency measures and adaptive filtering methods have already been proposed by Leclerc [5], [6]. Evaluation of these methods needs to synthesize sEMG data assuming TVD models between channels (an inverse sinusoid model and a sigmoid model in [5] and [6] respectively). In such cases, the TVD model choice is not critical since the investigated methods are independent of this model. However, the obtained performance suffers from high noise levels.

Optimal TVD estimators can be derived with Maximum-Likelihood estimation (MLE) method. Nevertheless, this kind of approach cannot be directly used because the MLE method leads to an optimization problem in an N-dimensional space where N is the number of recorded samples: a time-delay value has to be estimated for each time value and N parameters have to be estimated. An alternative way with the MLE method is to drastically reduce the parameters number to estimate. In this work, we chose to model the TVD with a polynomial function with p parameters. This can be done thanks to the Weierstrass theorem that ensures that any continuous compactly defined function can be arbitrarily approached by a polynomial function. So, instead of estimating the delay at each time instant, only p coefficients need to be estimated. A precise TVD function estimation can be estimated with a high value of p. At this stage, the compromise between the p order value and the com-

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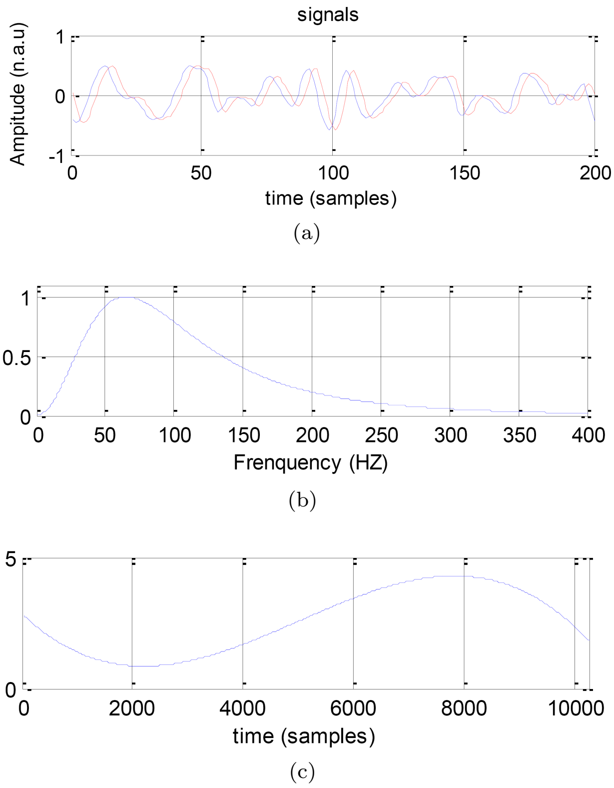


Fig.1:: (a) Synthetic sEMG signal (blue) and its delayed version with free noise (red). (b) Normalized PSD (c) Time varying delay with third degree polynomial model, $\Theta = [2.8627, -4.1246, 2.4526, -0.3337]$.

putational cost has to be considered. A low value of p implies a mismatch error between the Model and the true TVD function. A high value of p suffers from exponential computational cost and from convergence problems. For these reasons, we evaluate the opportunity to cut the delay function into many consecutive slices where linear or parabolic estimations could be sufficient. Results will be expressed through Monte Carlo simulations in terms of root mean square errors (RMSE) of these estimators as functions of the parameters (models of TVD, methods, lengths of the slices).

The paper is organized as follows. In section 2, the models of signals and TVD will be defined. In section 3, the MLE method and Phase coherency method for time varying delay in the case of two channels will be derived. Section 4 presents the simulation results with first synthetic sEMG data which follow the sinusoidal and polynomial models and second with real data. In section 5, we conclude the paper.

2. MODELS OF TIME-VARYING DELAY AND SEMG SYNTHETICS SIGNALS

Considering the sEMG signal $s(n)$ propagation between channel 1 and channel 2, a simple analytical model of two observed signals $x_1(n)$ and $x_1(n)$ in a discrete time domain, without shape differences, is the following:

$$\begin{aligned} x_1(n) &= s(n) + w_1(n) \\ x_2(n) &= s(n - \theta(n)) + w_2(n) \end{aligned} \tag{1}$$

,where $\theta(n)$ is the propagation delay between the two signals, and $w_1(n)$ are $w_2(n)$ assumed to be independent, white, zero mean, additive Gaussian noises, of equal variance σ^2. Once $\theta(n)$ is estimated, the MFCV can simply be deduced by $MFCV(n) = \frac{\Delta e}{\theta}$ where Δe stands for the inter-electrode distance, which is taken as 5 mm in the following. The digitization step is processed at the sampling frequency $F_S = 2048$ Hz. We detail below the two models used for the TVD function as well as the way for generating synthetic sEMG signals with predefined TVD functions.

2.1 Inverse sinusoidal model

In this study, we used the inverse sinusoidal model of TVD defined as follows:

$$\theta(n) = F_S \frac{5 \cdot 10^{-3}}{5 + 3sin(\frac{0.2n2\pi}{F_S})} \tag{2}$$

This model has been previously proposed in [9]: It takes into account reasonable physiological variations of MFCV that may be encountered during dynamical exercise situations.

In particular, the minimum and maximum MFCV values are 2 m.s^{-1} and 8 m.s^{-1} respectively. The maximum acceleration value is 2.5 m.s^{-2}. One period of the sine wave is considered corresponding to 5 seconds observation duration or to equivalently 10000 data samples.

2.2 Polynomial model

The TVD may be decomposed up to order p on the canonical polynomial basis as:

$$\theta(n) = F_S \sum_{k=0}^{p} \theta_k n^k \tag{3}$$

The TVD is thus defined by a $p+1$ dimensional vector with parameters $\Theta = [\theta_0 \theta_1 \cdots \theta_p]$. With 4^{th} order polynomial model, we have

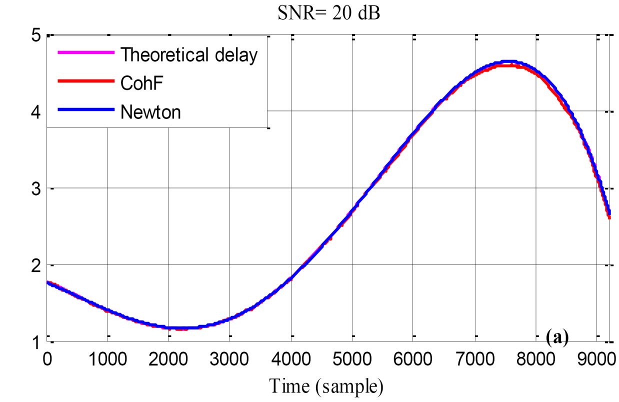
$$\begin{aligned} \theta(n) &= F_S(1.9125 - 0.3475n + 0.9366n^2 \\ &\quad -0.7187n^3 - 0.1051n^4). \end{aligned} \tag{4}$$

2.3 Delayed signal generation

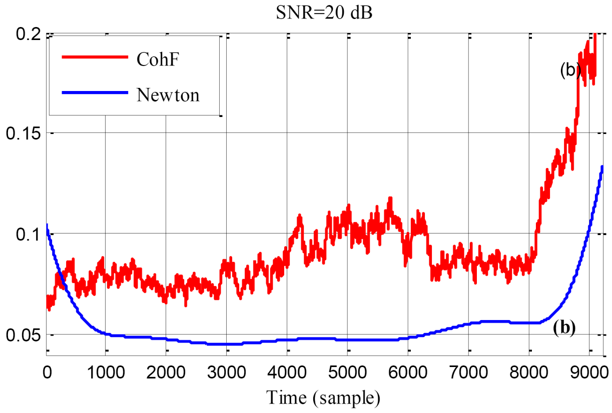
The signals are synthetic ones and are generated according to the following analytical Power Spectral Density (PSD) shape [9]:

$$PSD(f) = \frac{k f_h^4 f^2}{(f^2 + f_1^2)(f^2 + f_1^2)^2} \tag{5}$$

An example of sEMG PSD shape is given on [3] where the low and high frequency parameters are fixed as

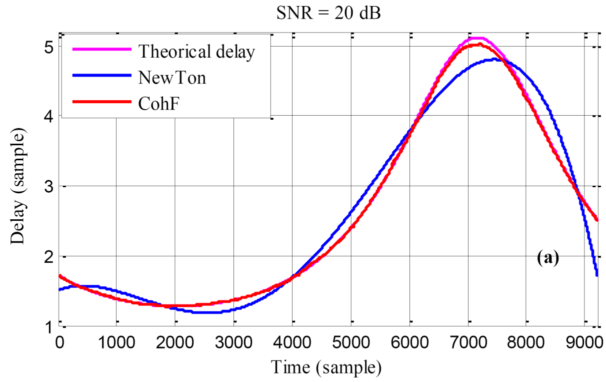


(a)

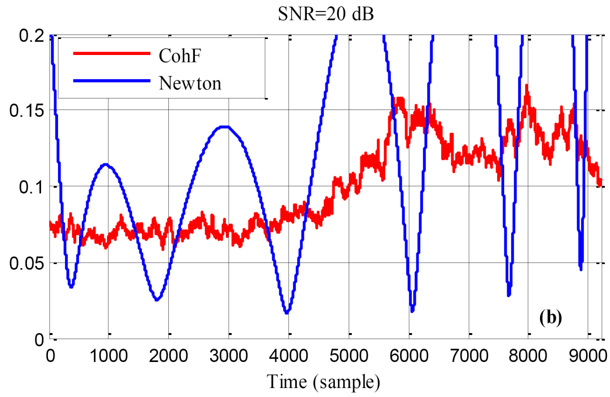


(b)

Fig.2:: TVD (a) and RMSE (b) as a function of time; Theoretical TVD as a 4^{th} order polynomial model (black) and its averaged estimation by the Phase Coherency method (red) and Newton method (blue). The parameters were: $\Theta_4 = [1.9125, -0.3475, 0.9366, -0.7187, -0.1051]$



(a)



(b)

Fig.3:: TVD (a) and RMSE (b) as a function of time; Theoretical TVD as an inverse sinusoidal model (black) and its averaged estimation by the Phase Coherency method (red) and Newton method (blue).

f_l =60 Hz and f_h =120 Hz respectively. The parameter k is a normalization factor.

The first channel is generated by linear filtering a white Gaussian noise with the impulse response corresponding to this PSD (i.e. the inverse Fourier transform of the square root of the previous PSD shape). Once the first channel is generated, its delayed version is created thanks to the sinc-interpolator [8]:

$$s(n-\theta(n)) = \sum_{i=-p}^{p} sinc(i-\theta(n))s(n-i) \quad (6)$$

The parameter p is the filter length and is fixed to p=40. Finally, both channels are distorted by adding white noise at a given signal to noise ratio (SNR) level. Fig. (1a) shows the first 200 samples of a 10000 samples synthetic sEMG realization and its delayed version at 20 dB. The delay law is a third order polynomial function shown in Fig. (1c).

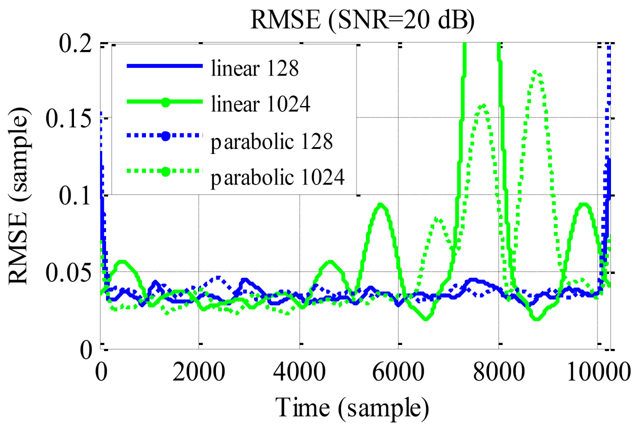


Fig.4:: RMSE as a function of time; Newton method with linear estimation by 128 and 1024 samples slices (blue and green, respectively); Newton method with parabolic estimation by 128 and 1024 samples slices (dotted blue and dotted green, respectively).

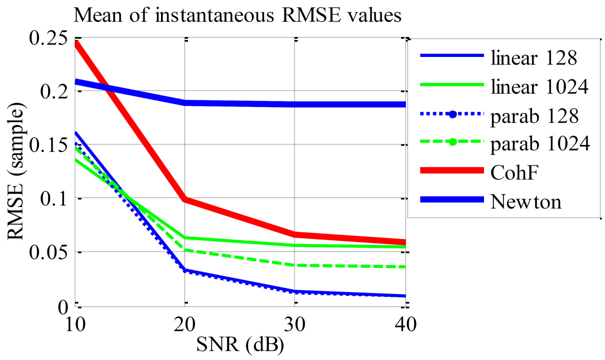


Fig.5:: RMSE mean values as a function of the SNR values. Newton method with linear estimation by 128 and 1024 samples slices (blue and green, respectively); Newton method with parabolic estimation by 128 and 1024 samples slices (dotted blue and dotted green, respectively); phase coherency method (fat red line); Newton method with 4^{th} order approximation (fat blue line).

3. METHODS

We detail below the MLE method followed by the Fourier phase coherency method for comparison purpose.

3.1 Maximum likelihood estimation (MLE)

Considering the model and the assumption of independent and Gaussian noise of $w_1(n)$ and $w_2(n)$, the estimation of the delay in the sense of maximum likelihood is obtained by maximizing the log-likelihood function, which is the probability density function of the observed data and the parameters to be estimated. The likelihood function is therefore defined by:

$$\Lambda(x_1, x_2; \theta; s) = p(x_1, x_2; \theta; s) = \Pi_{i=1}^{2} p(x_i(n); \theta) \tag{7}$$

$$= \left((2\pi\sigma^{\frac{N}{2}})\right)^2 exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{n=1}^{N}(x_1(n) - sn2 + n)\right]\right\} = 1Nx2n - \theta n - sn2$$

The log-likelihood function only keeps the parameters to be estimated writing as:

$$ln\Lambda(x_1, x_2; \theta; s) = -\sum_{n-1}^{N}(x_1(n) - s(n))^2 \tag{8}$$

$$-\sum_{n=1}^{N}(x_2(n-\theta(n)) - s(n))^2$$

The first derivative of the log-likelihood function with respect to $s(k)$ is

$$\frac{\partial ln\Lambda(x_1, x_2; \theta; s)}{\partial s(k)} = 2[x_1(k) - s(k)] \tag{9}$$

$$+2[x_2(k-\theta(k)) - s(k)]$$

The maximum of the log-likelihood function is obtained by setting this last expression to zero which leads to:

$$\hat{s_k} = \frac{x_1(k) + x_2(k-\theta(k))}{2} \tag{10}$$

Substituting $s(n)$ by $\widehat{s(n)}$ in the expression of log-likelihood function (7), we obtain:

$$ln\Lambda(x_1, x_2; \theta; s) = -\frac{1}{2}\sum_{n=1}^{N}(x_2(N-\theta(n)) - x_1 x_2) \tag{11}$$

Maximize the log-likelihood function is equivalent to minimizing the following expression:

$$\widehat{\Theta} = argmin\ e_t^2(\Theta) \tag{12}$$

Where

$$e_t^2(\Theta) = \sum_{n=1}^{N}(x_2(n-\theta(n)) - x_1(n))^2 \tag{13}$$

The problem of estimating $\theta(n)$ is the same as estimating the N-dimensional vector $\Theta = [\theta(1)\theta(2)\cdots\theta(N)]$. In the case of TVD polynomial models, this problem, as expressed in Equation (3), reduces to the estimation of a *p+1* dimensional vector $\Theta = [\theta(0)\theta(1)\cdots\theta(p)]$. In this work, we used the Newton method for the minimization of the function $e_t^2(\Theta)$.

3.2 Fourier phase coherency

The Fourier transform of any delayed signal introduces a new linear phase term. The temporal delay value may be simply retrieved by measuring linear slopes in the phase representation of the Fourier transform as a function of the frequency. The Fourier phase coherency method is an extension of this concept in the nonstationary case. It is based on the local Fourier coherence of two signals $x_1(t)$ and $x_2(t)$ that can be defined as [6]:

$$ConF(t,f) = \frac{E_t\{P_{x_1x_2}(t,f)\}}{\sqrt{E_t\{P_{x_1x_1}(t,f)P_{x_2x_2}(t,f)\}}} \tag{14}$$

Where $P_{x_1x_2}(t,f) = X_1(t,f)X_1^*(t,f) = |P_{x_1x_2}(t,f)|$ $e^{i\phi_{x_1x_2}(t,f)}$ is the local cross spectrum. $X_1(t,f)$ and $X_2(t,f)$ are the local Fourier transforms of the signals $x_1(t)$ and $x_2(t)$:

$$X_i(t,f) = \int_{-\infty}^{\infty} h(\tau - t)x_i(\tau)e^{i2\pi f\tau}d\tau \tag{15}$$

The function $h(t)$ is the Hanning weighting window function that restricts the Fourier transform around the time instant t. The asterisk refers to the conjugate of the signal. The expectations E_t are estimated by the Welch method. Each N-samples window is divided in three $\frac{N}{2}$ samples Hanning weighted windows with 50% of overlapping. It can be shown that

$$P_{x_1x_2}(t,f) \approx P_{SS}(t,f)e^{-2\pi f\theta(t)} \tag{16}$$

Since all the other terms in the coherence function are positive and real, the phase term in *CohF(t,f)* entirely contains at each time instant the delay $\theta(t)$.

4. RESULT AND DISCUSSION

A Monte-Carlo simulation with 100 independent runs was performed for each signal to noise ratio (SNR) value in order to study the noise impact of these estimators. In this work, two synthetic sEMG signals have the same value of SNR = 10, 20, 30, 40 dB respectively. Duration of the signals is 5 seconds.

4.1 Polynomial model

The parameters of the polynomial TVD (3) were fixed so that they fit the inverse sinusoidal TVD (2) in the mean square error sense. The obtained parameters were

$$\Theta_4 = [1.9125, -0.3475, 0.9366, -0.7187, -0.1051]$$

The 4^{th} order was used in order to show the model mismatch rather than the 3^{th} order. Synthetic data were generated using this polynomial TVD parameter set between the two channels. Fig. (2a) shows the TVD estimations using Newton method and Phase Coherency method. Fig. (2b) shows the root mean square errors (RMSE) of these estimators calculated by Monte Carlo simulations. It can be noticed that the Newton method gives better results with respect to the Phase coherency method (except at the beginning of the signals). This result was expected because the Newton method seeks for a polynomial model which is polynomial by construction. On the contrary, the Phase coherency method does not take into account the model of the delay.

4.2 Sinusoidal model

In this case, the TVD was the inverse sinusoidal model expressed in equation (2). Fig. (3a) shows the TVD estimations using the Newton method and the Phase Coherency method. The Newton method is based on a 4^{th} order polynomial TVD estimation which mismatch with the inverse sinusoidal model can clearly be observed. Fig. (3b) shows the RMSE of these estimators calculated by Monte Carlo simulations. In this case, the phase coherency method outperforms the Newton method except in the small temporal slices where the polynomial model matches the true TVD. In conclusion for this experimental setup, the RMSE values are principally due to model mismatches rather than estimation errors.

A possible way to improve the estimation is to increase the polynomial order in such a way that it reduces the matching model errors. However, this approach has two drawbacks:
Since the TVD variations cannot be a *priori* known, the appropriate order of the polynomial function cannot be chosen. Estimating the appropriate model order is a difficult task.
A high order polynomial function guaranties a good matching between the true delay and the model function but at a prohibitive computation cost. When the polynomial order increases, the Newton optimization method becomes intractable.

The proposed issue for this problem is to consider the whole observation as the succession of temporal slices for which low order polynomial functions are sufficient for a correct matching. We tested linear and parabolic function models. The lengths of each slice were chosen as 128 and 1024 samples respectively. The linear and parabolic function parameters were estimated for each slice independently by Newton method, respectively. Then the estimated functions were stuck to each other. Due to the discontinuity of the TVD after the reconstruction, a low pass filter, zero phases, and 300 order with *cutoff* frequency equal 3 Hz, was used.

Fig. (4) shows the RMSE as a function of time for the Phase Coherency method, and for the Newton method considering linear or parabolic estimations by slice, at a SNR=20 dB. The results show that a small slice length (128 samples) is preferable than a longer one (1024 samples). Moreover, the parabolic approach does not seem to significantly improve the results compared with the linear approach.

We carried out further experiments by evaluating the noise impact on the results. In order to get more concise results, we considered the mean RMSE indicator that corresponds to the whole signal RMSE averaging. Fig. (5) displays these results as a function of the SNR values. The Coherence Phase method and the Newton method are also displayed for comparison.

Again, the shorter the slice length, the better the results (thin blue curves). This is true for high SNR values. In the case of strong noise (SNR=10 dB), the results are quite the same whatever the slice length: the 128 samples number makes the model parameters estimation more sensitive to noise than the 1024 samples number. Using parabolic model instead of linear model becomes to be interesting for high SNR values and for long slices.

The essential result is the gain obtained by the slicing strategy with respect to the 4^{th} order polynomial modeling. While the RMSE is around 0.2 samples for 10 to 40 dB SNR values, the RMSE decreases from 0.15 samples at 10 dB to 0.01 samples at 40 dB with the 128 samples linear slice estimation strategy. The RMSE with the Coherence Phase strategy follows the same decreasing tendency with the SNR value but with around 0.05 RMSE samples more. In conclusion, the modeling mismatch with the true TVD can be bypassed with a low order polynomial modeling of the data being sliced. This strategy can advantageously be applied to any TVD continuous evolution.

5. CONCLUSION

Maximum-likelihood estimation combined with Newton method for time varying delay estimation was applied to synthetic sEMG signals to estimate the conduction velocity. We first proposed to approximate an inverse sinusoidal TVD with a 4^{th} order polynomial model for the synthetic signals. Secondly, we proposed to cut the TVD into many slices (with linear or parabolic approximations) and thus the TVD was estimated by slices. The proposed method improves the delay estimation with a gain of at least 0.05 sample precision by comparison with the classical MLE approach and with the Phase Coherency method [6]. The results of this approach are very encouraging regarding the delay values range varying between 1.2 samples up to 2 samples.

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