Ultrasonic Diffraction Tomography

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ABSTRACT

Diffraction tomography is a technique for imaging with acoustic fields in which parameter, such as reflective index, sound velocity, etc., can be mapped from scatter wave resulting from insonifying the object with a plane wave at a single temporal frequency. By solving the direct scattering problem, the scattered field can be presented in term of scattering parameters. Different inversion techniques can be applied to express takes advantage of the linearization process of the non-linear wave equation describing wave propagation in heterogeneous media under for limited class of scattering. Specifically, when the scattering effect is weak, one can invoke the Born or Rytov approximation and thus derive the generalized Fourier Slice Theorem to reconstruct the cross-section of the insonified object. Although diffraction tomography is a promising technology for medical application as it provides a quantitative ultrasonic image, its realization toward medical use is still far-to-go, this may be due to the complexity of the hardware involved. In this research we investigate a potential use of diffraction tomography for medical application by using a delicate-designed ultrasonic computerized tomographic system. The result of experiment investigation of diffraction tomography is very promising.

Keywords: Ultrasound; Tomography; Diffraction

1. INTRODUCTION

Ultrasound has potentially many important technological applications. These include medical imaging [1], nondestructive testing [2], geophysics [3], and robotic vision [2]. The advantages of microwave and/or ultrasound imaging offered over more conventional imaging are numerous. They include the relatively low health hazard of non-ionizing, low power of sources, such as microwave, ultrasound, etc., its ability to image physiological properties of a tissue or organ, and the likely cost competitiveness of the imaging equipment.

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In ultrasound imaging, conventional B-scan image use pulse echo ultrasound reflected from tissue interface to form image as tomography. Pulse-echo B-scan image is not quantitative imaging. Work is now progress on methods of correlating (quantitatively) these scattered returns with local properties of tissue [4-5]. This is made difficult by the fact that the scattered returns are modified every time they pass through an interface; hence the interest in ultrasonic computed tomography as an alternative strategy for quantitative imaging with sound. This is mainly due to the fact that ultrasonic computed tomography (UCT) may generate cross-sectional images (tomograms) of three different material properties: (i) an attenuation tomogram representing the ultrasonic energy loss due to scatter and absorption in the material; (ii) a speed-of-sound tomogram representing a measure of the elastic constants in the material [6]; and, (iii) a reflection tomogram representing a map of ultrasonic impedance mismatch from boundaries and inhomogeneities. As a result, ultrasonic computed tomography now receives an intensive attention from many researchers.

There is a fundamental difference between ultrasonic computed tomography and x-ray computed tomography. X-ray, being a high-energy electromagnetic wave, can travel in a straight line and hence the projection data can derive form the line integral of a function of attenuation coefficient of the object along the traveling straight line. Unlike x-ray tomography, the diffraction and refraction of the ultrasonic wave while it gets through the interfaces of the tissue having different reflective indexes make ray travel not in the straight line, hence the line integral geometry as in the X-ray case cannot be implemented here. Under the assumption that the soft tissue has little or no change in the refractive indexes, however, one can still use the straight line geometry. In such case, the traditional filtered back-projection or algebraic reconstruction algorithm for non-diffraction tomography should provide the approximated results [wscg2004].

The difficulty with ultrasound imaging is associated problem with performing object reconstruction. Because ultrasound imaging experience significant attenuation, scattering problem and diffraction, standard tomographic reconstruction schemes are not readily applicable and are of only limited usefulness. In an attempt to overcome this problem, many approaches to ultrasound imaging has been investigated in the recent past[13]. Ultrasonic Diffraction tomography is a technique for inverting the differential wave equation governing interaction between the insonifying field and the scattering medium. In the presence of weak scattering effect, one can take advantage of the linearization process of the wave equation obtainable by invoking either the Born [1] and the Rytov [1] approximation to perform the inversion algorithm and thus derive the generalized Fourier Slice Theorem. The advantages of this approach is that it is comparatively straightforward to apply and usually computationally efficient. These algorithms can be implemented on small computers in connection with imaging systems using ultrasound [14-16] for quasi-real time processing purposes. Although diffraction tomography is a promising technology for medical application, its realization toward medical use is still far-to-go, this may be due to the complexity of the hardware involved. In this paper, we investigate a potential use of diffraction tomography by using the delicate-designed UCT system.

The paper is organized as follows. Section 2 presents mathematical models of the transmission mode ultrasonic pulse and the determination of the integrated refractive index from time delay and phase shift. Section 3 shows the acquisition system for gathering the projection from specimen. Section 4 is the experimental results tested on 2 different phantoms showing the reconstructed function of refractive index and sound velocity, including the calculation of error. Section 5 is the discussion and conclusion.

2. DIFFRACTION TOMOGRAPHY

When an object is insonified with a plane wave as shown in Fig. 1, the propagation of acoustic waves can be modeled with the Helmholtz's wave equation. For a temporal frequency of radian per second, a field, $\phi(\vec{r})$ satisfied the equation [ref]

$$(\nabla^2 + k_0^2)\phi_s(\vec{r}) = -o(\vec{r})\phi(\vec{r}), \qquad (2.1)$$

where

$$o(\vec{r}) = k_0^2[n^2(\vec{r}) + 1], \qquad (2.2)$$

$$\phi(\vec{r}) = \phi_0(\vec{r}) + \phi_s(\vec{r}), \qquad (2.3)$$

$$k_0 = 2\pi/\lambda, \qquad (2.4)$$

and where

$o(\vec{r})$ is the object function,
$\phi(\vec{r})$ is the total field,
$\phi_s(\vec{r})$ is the scattered field,
$\phi_o(\vec{r})$ is the incident field,

$n(\vec{r})$ is the refractive index,
k_0 is the wave number,
λ is the wavelength,

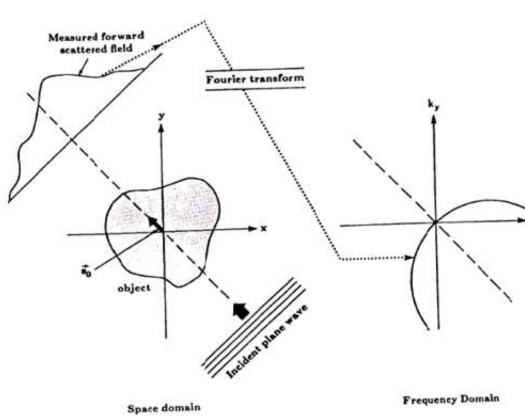


Fig.1: *Fourier Diffraction Theorem*

Helmholtz equation or inhomogeneous wave equation cannot be solved directly, but solution can be written in term's of a Green's function ,$g(\vec{r}|\acute{r})$,

$$\phi_s(\vec{r}) = \int g(\vec{r} - \vec{r}')o(\vec{r}')\phi(\vec{r}')d\vec{r}' \qquad (2.5)$$

$$g(\vec{r}|\vec{r}') = \frac{j}{4}H_0^{(1)}(k_0 R) \qquad (2.6)$$

$$R = \vec{r} - \vec{r}' \qquad (2.7)$$

where H_0^1 is the Hankel Function of the first kind. Solving the object function $o(\vec{r})$ when the incident field $\phi_o(\vec{r})$, scattered field $\phi_s(\vec{r})$ and hence the total field $\phi(\vec{r})$ are known is called the inverse problem. However, solving (2.5) is not a straightforward as it is a non-linear equation. Two methods, due to Born and Rytov, are commonly used to approximate the solution. For the first Born approximation, the scattered field $\phi_s(\vec{r})$ is assume to be small compared with $\phi_o(\vec{r})$; then equation (2.5) becomes

$$\phi_s(\vec{r}) = \int g(\vec{r} - \vec{r}')o(\vec{r}')\phi_o(\vec{r}')d\vec{r}' \qquad (2.8)$$

which is no longer non-linear. By substituting Hankel Function of the first kind function [Mor53]

$$H_0^1(k_0|\vec{r} - \vec{r}'|) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\beta}e^{j[\alpha(x-x')+\beta(y-y')]}d\alpha \quad (2.9)$$

into Green's function, equation (2.8) can be written as

$$\phi_s(\vec{r})=\frac{j}{4\pi}\int o(\vec{r}')\phi_o(\vec{r}')\int_{-\infty}^{\infty}\frac{1}{\beta}e^{j[\alpha(x-x')+\beta(y-y')]}d\alpha d\vec{r}' \quad (2.10)$$

where β
Let the incident wave propagates along the positive y axis; then

$$\phi_0(\vec{r}) = e^{j\vec{s}_o\cdot\vec{r}} \quad (2.11)$$

where $\vec{s}_o = (0, k_0)$
Let the scattered fields are measured by a linear array detector at $y = l_0$, then , and $u_0(\vec{r})$with substituted, equation (2.10) becomes

$$\phi_s(x, y = l_0) = \frac{j}{4\pi}\int_{-\infty}^{\infty} d\alpha.$$

$$\int_{-\infty}^{\infty}\frac{0(\vec{r})}{\beta}e^{j[\alpha(x-x')+\beta(l_o-y')]}e^{jk_oy'd\vec{r}'} \quad (2.12)$$

The inner integral is recognized as the two-dimensional Fourier transform of the object function evaluated at a frequency of $(\alpha, \beta - k_0)$. Then we can write

$$\phi_s(x, y = l_0) = \frac{j}{4\pi}\int_{-\infty}^{\infty}\frac{1}{\beta}e^{j(\alpha x+\beta l_o)}O(\alpha - \beta - k_0)d\alpha. \quad (2.13)$$

where O denoted the two-dimensional Fourier transform of the object function. Let $\Phi_s(\omega, l_0)$ denote the one-dimensional Fourier Transform of the scattered field, $u_s(x, l_0)$, with respect to x, that is

$$\Phi_s(\omega, l_0) = \int_{-\infty}^{\infty}\phi_s(x, l_0)e^{-j\omega x}dx \quad (2.14)$$

Substitute (2.14) into (2.13) and using property of Fourier transform, we can have

$$\Phi_s(\omega, l_0) = \frac{j}{2\sqrt{k_o^2 - \alpha^2}}e^{j\sqrt{k_0^2-\alpha^2}l_o}.$$

$$O(\alpha, \sqrt{k_o^2 - \alpha^2} - k_0) \quad (2.15)$$

Equation 2.15 provides the relationship between one dimensional Fourier transform of scattered field and two dimensional Fourier transform of object function which leads to generalized Fourier slice theorem. In general, when an object is insonified with a plane wave as shown in Fig. 1, the Fourier transform of the forward scattering fields measured in a line perpendicular to the direction of propagation of the wave gives the values of the 2D Fourier Transform of the object along a circular arc as shown in the figure 1. By illuminating an object in many different directions and measuring the diffracted projection data, one can fill up the Fourier space with the samples of the Fourier transform of the object over ensemble of circular arcs and then reconstruct the object by Fourier inversion

3. ACQUISITION SYSTEM

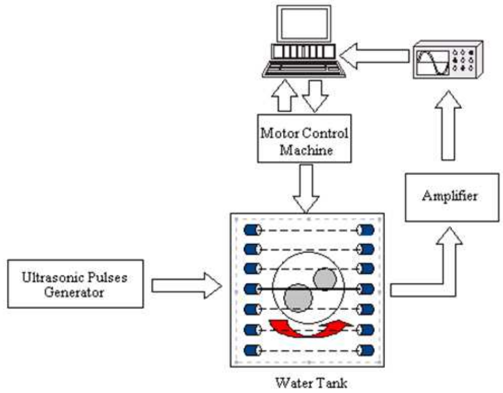


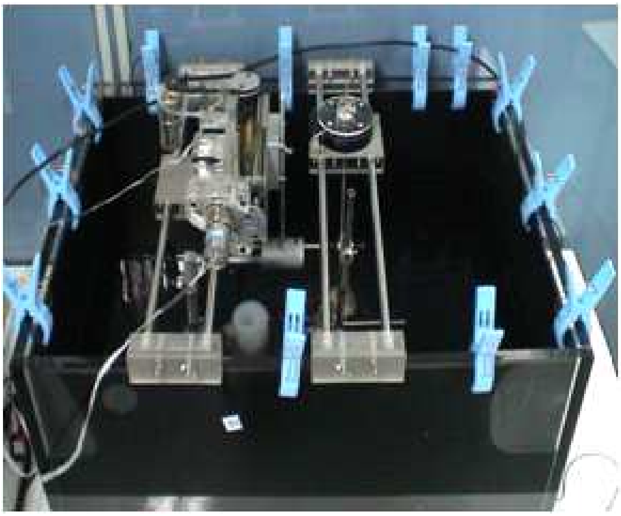
Fig.2: *Diagram of the acquisition system*

The acquisition system shown in figure 2 consists of

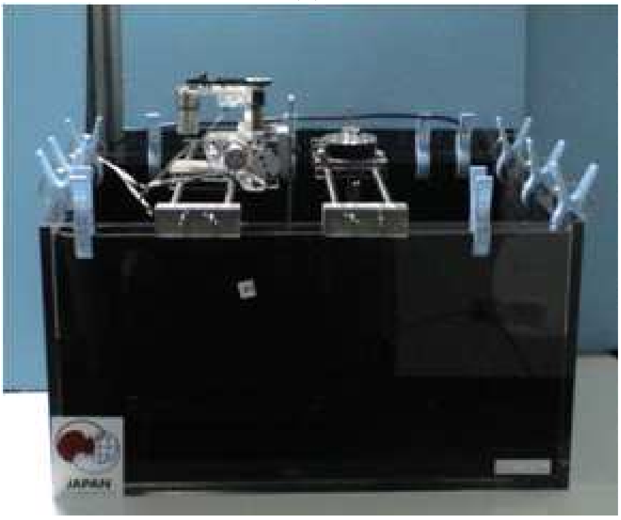
- Specimen's platform capable of moving in the azimuthal direction.
- Water tank used to immerse the specimen. The water helps to couple the signal between the transmitter,the specimen, and the receiver.
- Ultrasonic transmitter and receiver having the center frequency at 3.75MHz. Both are attached to the mechanic arms moving in the horizontal direction.
- Ultrasonic pulse generator which generates the broadband pulse to the transmitter.
- Signal amplifier amplifying the received signal.
- A/D Converter converting the analog signal to digital signal with the sampling rate of 500 MHz.
- Absorber covering the internal wall of tank to prevent the echo of sound.

The prototype of the system is shown in figure 3.

In order to keep the projection data, the specimen is rotated to the specified angle and then the ultrasonic transmitter and receiver are moved to the 1st projection position. The pulse generator generates ultrasonic broadband pulse to the transmitter, and then the receiver receives the distorted pulse. The pulse is transformed to the digital format by the A/D converter and stored in the computer. When the transmitter and receiver move to the next projection position, the same process starte again. After complete one projection, the specimen is rotated to the new angle and all the processes restarte. In the experiment, the interval of angle is set to 10 degrees, and the interval or resolution between each scanning point is 1 millimeter. Figure 4 shows two types of pulses; the first of which is the reference pulse or $y_w(t)$ collected from sending the pulse through the water and the second is the distorted pulse or $y(t)$ collected from sending the pulse through the specimen.



(a)

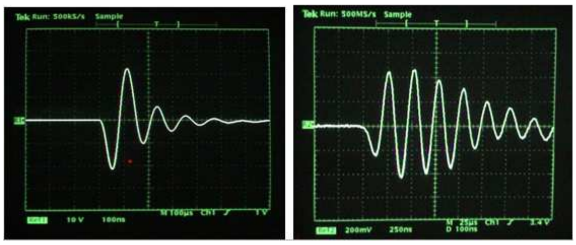


(b)

Fig.3: *The prototype of ultrasonic acquisition system (a) top view, (b) side view.*

4. EXPERIMENTAL RESULTS

Two phantoms with different patterns remarked as "A" and "B" are used to evaluate the reconstruction process. The phantoms are made from the gelatin having the refractive index close to the water, and the diameter of each phantom is approximately 60 millimeters.



(a) (b)

Fig.4: *Two types of pulses, (a) the reference pulse $y_w(t)$ and (b) the distorted pulse $y(t)$.*

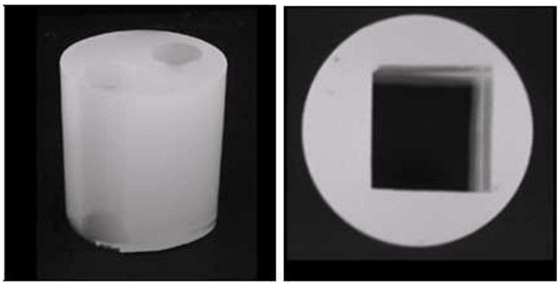


Fig.5: *Phantom A, side view and top view.*

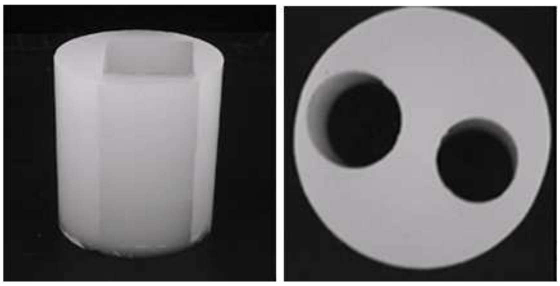


Fig.6: *Phantom B, side view and top view.*

Two methods are used to solve for the integrated refractive index, the time delay and the phase shift. In the time delay method, T_d is detected from the maximum peak of the broadband pulses whereas in phase shift method, T_d is calculated from equation 7 after taking Fourier transform of the reference and distorted signals. When the set of projections is found, the filtered backprojection with the Hann filter is employed to find the cross-sectional function. By using equation 6 and equation 5, the reconstructed function of refractive index and the reconstructed function of sound velocity can be obtained respectively. The results from time delay are shown in figure 7 and 8, and the results from phase shift at selected frequencies are shown in figure 9 and 10. Table 1 and 2 correspond to the Mean Square Error of phantom A and B respectively. The formula of MSE is given by

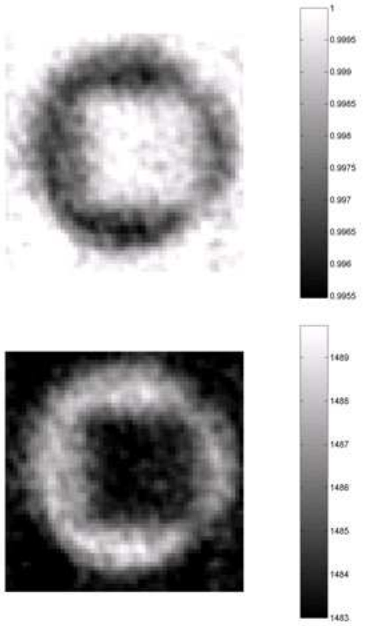


Fig.7: *Reconstruction of (a) refractive index and (b) sound velocity of phantom "A" using time delay.*

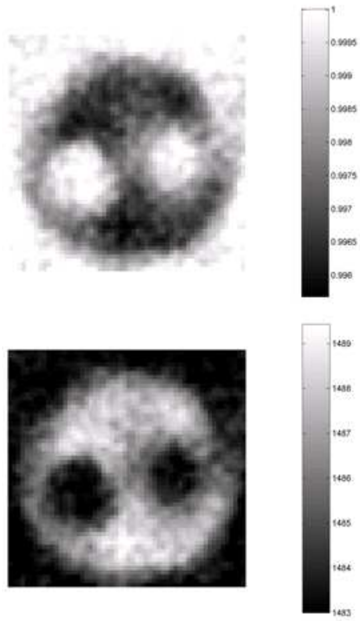


Fig.8: *Reconstruction of (a) refractive index and (b) sound velocity of phantom "B" using time delay.*

$$MSE = \frac{\int\int \left[o(x,y) - o'(x,y)\right]^2 dxdy}{\int\int \left[o(x,y)\right]^2 dxdy} \times 100 \qquad (8)$$

where $o(x, y)$ denotes the value of the original function, $o'(x, y)$ denotes the value of the reconstructed function, both of which are normalized to 0-255. The MSE of both phantoms at different frequencies are also presented in the graphs.

Table 1: *Mean Square Error of Phantom "A"*

Method		Mean Square Error (%)
Time Delay		29.405
Phase Shift at Frequency (MHz)	1.50	30.072
	2.00	26.679
	3.00	25.335
	3.50	25.130
	3.70	25.447
	3.75	**25.308**
	4.00	25.794
	4.25	26.275
	6.oo	29.061

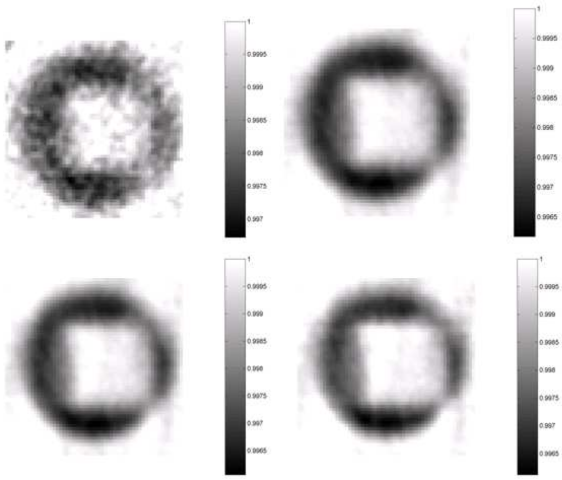


Fig.9: *Reconstruction of the refractive index of phantom "A" using phase shift at frequency of 1.5 MHz, 3.5 MHz, 3.75 MHz, and 6MHz respectively.*

Table 2: *Mean Square Error of Phantom "B"*

Method		Mean Square Error (%)
Time Delay		24.217
Phase Shift at Frequency (MHz)	1.50	28.737
	2.00	23.774
	3.00	22.022
	3.50	22.318
	3.70	21.759
	3.75	**21.259**
	4.00	21.428
	4.25	21.449
	6.oo	23.192

5. DISCUSSION AND CONCLUSION

We investigated the quantitative ultrasonic imaging using the diffraction tomography for heterogeneous object. The object to be image is insonified by a plane wave and the transmitted ultrasonic pulsed is measured along a line perpendicular to the direction of propagation of the incident ultrasonic wave. We assume the weak scattering of the ultrasonic wave as it traverses through the object. The cross-section of the

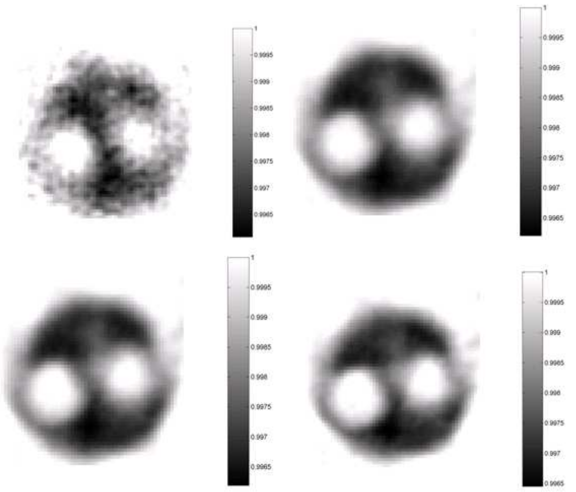


Fig.10: *Reconstruction of the refractive index of phantom "B" using phase shift at frequency of 1.5 MHz, 3.5 MHz, 3.75 MHz, and 6MHz respectively.*

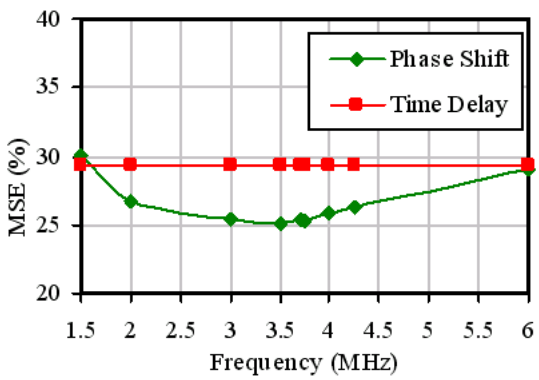


Fig.11: *Plotting of the MSE of phantom "A"*

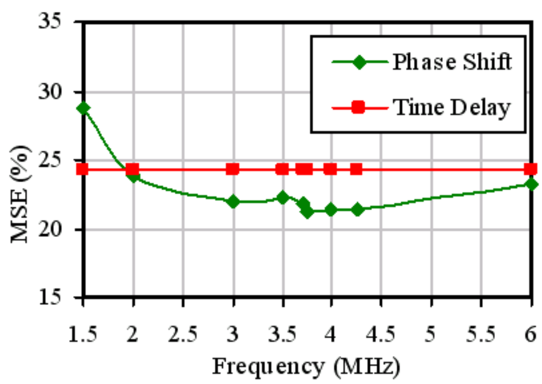


Fig.12: *Plotting of the MSE of phantom "B"*

object is derived from generalized Fourier slice theorem. The experiment results indicate that the diffraction tomography method provides quantitatively accurate imaging. Despites its promising technique, diffraction tomography is subjected to various limitations, which include artifacts due to diffraction effects in strong inhomogenous media and a quantitative reconstruction is only possible for a non-applicable restrictive domain in biomedical application due to the fact that Born and Ritov approximations are , in general, not valid. Further study is required to evaluate the performance of the diffraction tomography using a more realistic method.

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